Front-End Analysis

Matt Yubas

Topic: Algebra 1
Instructional Aim: PreAP 8th grade students will learn how to solve quadratic equations.

Needs Assessment

The purpose of this needs assessment is to evaluate an instructional module on solving quadratic equations for Pre-AP 8th grade students taking Algebra 1 at a middle school in Lubbock TX.

The Need

The Texas Education Agency (TEA) has created a curriculum for each grade level and each course. The objectives for each course are known as Texas Essential Knowledge and Skills or TEKS. The TEKS identify what students need to know and be able to do in each subject area.

For the given Instructional Aim, TEA states the following Algebra 1 TEKS with regards to solving quadratic equations:

> 8(A) Solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula.

Students will be assessed on solving quadratic equations on a local District Assessment and for the State of Texas Assessments of Academic Readiness (STAAR) End of Course Exam.

The Gap

Prior knowledge of solving quadratic equations needs to be understood. It's anticipated there will be a gap for this Instructional Aim. The TEKS Vertical Alignment document for mathematics shows students have not learned how to solve quadratic equations. To confirm this assumption, data will be collected with a pre-assessment to determine knowledge and skills of solving quadratic equations.

By the time this topic is introduced, students are expected to be competent with the characteristics of quadratic functions including calculating the axis of symmetry and vertex, as well as predicting the effects of parameter changes to the parabola. In addition, students are also reasonably competent with adding, subtracting, multiplying, and dividing polynomials of degree one and degree two.

The Solution

Middle school students are somewhere between concrete and abstract thinking. And solving quadratic equations is an abstract concept (Almarode, 2013). If the pre-assessment confirms there is a lack of knowledge and skill to solve quadratic equations, instructional intervention is needed.

Students could learn methods to solve quadratics on their own with online tutorials, but there are several steps and tricks that many 13 year old students with find difficult. Adult instruction is needed to scaffold students. In addition, given the curriculum time constraints, students require someone to guide them with concise steps and procedures and to offer immediate feedback. Therefore, instructional intervention will best serve the needs of these middle school students.

Almarode, J. (2013). *Captivate, Activate, and Invigorate the Student Brain in Science and Math.* Corwin.

Goal Analysis

Comprehensive Goal: 8th grade students in Pre-AP Algebra 1 will solve quadratic equations.

Goal Analysis:

- Solve quadratic equations using the quadratic formula.

    1. Identify the coefficients of a quadratic function
    2. Write the quadratic formula for a quadratic function
    3. Simplify inside the radical of a quadratic function using order of operations
    4. Form two equations from a quadratic formula in the form of x=
    5. Simplify each equation and solve for x
    6. Write the quadratic solution set found from the quadratic formula

- Solve quadratic equations with a graphing calculator.

    7. Identify the quadratic equation from a situation
    8. Power on and navigate the main menus of a graphing calculator
    9. Enter the quadratic function into the calculator
    10. View the graph of a quadratic function
    11. Find the solutions of a quadratic function using the graph
    12. Write the quadratic solution set found from the graph

Learner Analysis

The target audience for this Algebra 1 course are 8th grade Pre-AP students. The ages are 13 and 14. Genders are about an equal mix of boys and girls. Ethnic makeup is White, Hispanic, Asian, African American, and mixed race.

The students are highly self-motivated to learn and motivated by their parents to excel. They are goal oriented, generally quick to understand, and overall intelligent. Their parents are typically professionals including secondary teachers, professors, medical doctors, attorneys, scientists, and engineers.

General Characteristics

- Age: 13 – 14
- Gender: balanced mix of boys and girls
- Ethnicity: White, Hispanic, Asian, African American, and mixed race
- Language: English as a first language, or mostly fluent as a second language

Entry Behaviors

- Ability to score at least 70% on the STAAR 7th Grade Mathematics Assessment
- Experience with graphing calculators
- Higher than average reading scores
- Eager to learn and desire challenges
- Ability to stay organized and on track with lessons and homework
- Ability to understand English

Prior Knowledge

- 7th grade Pre–AP Math
- Introduction to functions
- Introduction to solving one–step equations
- Higher than average vocabulary

Motivation Level

- Self–motivated to learn
- Competitive with peers

Education and Ability Levels

- 8th grade Pre-AP
- Ability to solve multi-step processes on their own
- Ability to work independently

- Ability to work with abstract concepts

General Learning Preferences

- Prefer to learn topic background information and see how examples are worked out by direct instruction
- Like to see multimedia tutorials with explanations and examples
- Like to solve math problems independently
- Want to know topics are relevant in some way to the real-world or at least know how this fits as a high school prerequisite
- Want a teacher who is caring, organized, prepared, and knowledgeable

Learners with Disabilities

- A few students need extra time to process and complete their work
- A few students have mild anxiety issues
- A few students have mild hearing and visual impairments

Implications for instruction

- Pace of instruction can generally move rapidly
- Can provide large chunks of instruction at a time
- Go beyond the basics of quadratics and offer more abstract concepts
- Tie-in real-world examples such as time and height of throwing a football or basketball
- Show video tutorials such as those from Khan Academy and make available on teacher website
- Provide advanced graphing calculator instruction
- Provide independent work soon after instruction and examples
- Increase rigor and challenges
- Provide early feedback to make sure they are on track
- Can offer competitions to increase interest
- Students with learning disabilities need extra attention and extra time for assessments
- Some students have seating preferences related to physical impairments and anxiety conditions

Contextual Analysis

Data for a contextual analysis was collected from a survey, discussions, and observations. A survey was given to students with forced choice and open ended questions. The survey collected information on their course goals, what they want to learn, their interests, technology ability and availability, environmental preferences, and any personal issues.

After reviewing the data, there does not appear to be any limitations on instruction to meet the instructional aim.

Orienting Context

Students have a goal to earn either an A or B in this class. They are interested in challenges and want to learn. They are not 100% sure how this instruction will help them in the future. But they know the instruction will help their knowledge and skills as a prerequisite for Algebra 2. And students know they are accountable for the instructional aim since the assessments will count toward their semester grade.

The instruction needs to increase student's knowledge and skills as well as offer challenges. In addition, content and discussions should tie to real-world applications and how the topic feeds into Algebra 2. And instruction and assessments need to directly relate to learning objectives.

Instructional Context

The environment of the classroom requires normal lighting, temperature control, student desks, and minimal disruptions. Equipment needs to be prepared and checked before instruction. Equipment required includes: teacher computer and cables, network connection, Smartboard projector, Smart Response software, Smart Response clickers for each student, graphing calculators for each student, and copier for handouts.

Transfer Context

There needs to be opportunities for students to apply what they have learned. Students will be shown examples of solving quadratic functions during instruction. To check for understanding, students are given handout and asked to solve similar problems and submit results via a clicker. Students can use the aid of a graphing calculator. Additional problems to assess student transfer of knowledge and skills are provided. Students can work in pairs and or independently. After reviewing the results, students are assessed for a grade to measure depth of transfer.

Task Analysis

Comprehensive Goal: Solve a quadratic equation using the Quadratic Formula.

Task Analysis

1. Identify the coefficients.
    a. Look at the given quadratic equation.
    b. If the equation is not in the format: $f(x) = ax^2 + bx + c$, then rearrange terms using inverse operations.
    c. identify and write on paper the "a" coefficient, "b" coefficient, and "c" coefficient.
2. Write the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

    a. On paper write:
    b. Replace the variables "a," "b," and "c" with numerical values found in the quadratic equation.
3. Simplify inside the radical.
    a. Using the principals of order of operations, simplify the terms inside the radical.
    b. Find the square root of the value.
    c. Write: x = –"b" value ± square root value / divided by 2 times the "a" value.
4. Form two equations.
    a. Break the quadratic formula into two equations at the ± section.
    b. Write: x = –"b" value **+** square root value / divided by 2 times the "a" value.
    c. Write: x = –"b" value **–** square root value / divided by 2 times the "a" value.
5. Simplify each equation.
    a. Simplify the "+" equation and solve for x.
    b. Simplify the "–" equation and solve for x.
6. Write the solution(s).
    a. Write the solution(s) using braces.
    b. If there are two solutions, write the smaller numerical solution of x on the left side, followed by a comma, and larger numerical solution on the right side in the format "{smaller solution, larger solution}."
    c. If both solutions are the same number, there is only one solution, then write inside the braces the one numerical solution in the format "{solution}."
    d. If the number inside the radical is a negative number, there is no solution, then write "No Solution."

Comprehensive Goal: Solve a quadratic equation by graphing using a TI-nspire graphing calculator.

Task Analysis

7. Identify the quadratic equation.
    a. Look at the given quadratic equation.
    b. If the equation is not in the format: $f(x) = ax^2 + bx + c$, then rearrange terms using inverse operations.
    c. Write the quadratic equation on paper.
8. Power on the calculator by pressing the "on" button.
9. Enter the quadratic function.
    a. Press "1: New Document."
    b. If you see "Do you want to save 'Unsaved Document'?" select No.
    c. Press "2: Graphs."
    d. At the top of the screen where you see "f1(x)=" type the quadratic function in the form $ax^2 + bx + c$ and press the "enter" button.
10. View the graph.
    a. You will see a graph on your screen.
    b. If you do not see the graph on your screen, press the "menu" button, select "4: Window / Zoom" item, and select "4: Zoom – Out" item.
11. Find the solution(s).
    a. If the graph does **not cross the x-axis**, write "No Solution" on your paper
    b. If the graph touches the x-axis at **one point**, the solution is the vertex, then:
        i. Press the "menu" button, "6: Analyze Graph" item, and "1: Zero" item.
        ii. Place the "lower bound" cursor to the left of the vertex, and press the "enter" button, move the cursor to the right of the vertex with the touch-pad and press the "enter" button.
        iii. The "x" value of the displayed ordered-pair is the solution.
    c. If the graph crosses the x-axis at two points, there are two solutions, then:
        i. Press the "menu" button, "6: Analyze Graph" item, and "1: Zero" item.
        ii. Place the "lower bound" cursor to the left of the leftmost x-intercept and press the "enter" button.
        iii. Move the cursor to the right of the leftmost x-intercept with the touch-pad and press the "enter" button.
        iv. The "x" value of the displayed ordered-pair is the first solution.
        v. Press the "menu" button, "6: Analyze Graph" item, and "1: Zero" item.
        vi. Place the "lower bound" cursor to the left of the rightmost x-intercept and press the "enter" button.
        vii. Move the cursor to the right of the rightmost x-intercept with the touch-pad and press the "enter" button.
        viii. The "x" value of the displayed ordered-pair is the second solution.

12. Write the solution(s).
    a. Write the solution(s) using braces.
    b. If there are two solutions, write the smaller numerical solution of x on the left side, followed by a comma, and larger numerical solution on the right side in the format "{smaller solution, larger solution }."
    c. If there is only one solution, then write inside the braces the one numerical solution in the format "{solution}."
    d. If there is no solution, then write "No Solution."

Instructional Objectives

Comprehensive Goal: Solve a quadratic equation using the Quadratic Formula.

1. The learner will identify the coefficients of the quadratic equation.
2. The learner will describe the quadratic formula using the values of the coefficients.
3. The learner will calculate the value inside the quadratic formula radical using order of operations.
4. The learner will formulate the two equations from the quadratic formula from the positive and negative algebraic expressions.
5. The learner will simplify a quadratic equation and solve for an "x" value.
6. The learner will express the quadratic solution set, from the equations, using braces { } in the proper order, or indicate "No Solution."

Comprehensive Goal: Solve a quadratic equation by graphing using a TI-nspire graphing calculator.

7. The learner will arrange a quadratic equation in the format $f(x) = ax^2 + bx + c$.
8. The learner will identify the major calculator buttons and power on the calculator.
9. The learner will create a new document and type the quadratic function in the form "$f1(x) = ax^2 + bx + c$" into the calculator.
10. The learner will manipulate the graph of the quadratic function using the "Window / Zoom" function of the calculator.
11. The learner will determine the solutions of a quadratic function using the x-intercepts "Zero" function of the calculator.
12. The learner will express the quadratic solution set, from x-intercepts, using braces { } in the proper order, or indicate "No Solution."

Expanded Performance-Content Matrix

Objective Classification

Each number represents a corresponding objective.

Content	Performance	
	Recall	**Application**
Fact		
Concept	1	
Principals and rules		2
Procedure	8	3, 4, 5, 6, 7, 9, 10, 11, 12
Interpersonal		
Attitude		

Comprehensive Goal: Solve a quadratic equation using the **Quadratic Formula**.

1. The learner will identify the coefficients of the quadratic equation.

   For a quadratic function $x^2 - 2x - 24 = 0$, what are the coefficients? Fill-in a numerical value for each.

   a = ___

   b = ___

   c = ___

2. The learner will describe the quadratic formula using the values of the coefficients.

   The path that a football takes can be described by the equation $h = 15t - 5t^2$ where *h* is the height in meters of the football at time *t*, in seconds. What is the quadratic formula using the coefficients of this situation?

   A. $x = \frac{-15 \pm \sqrt{15^2 - 4(-5)0}}{2(-5)}$

   B. $x = \frac{-15 \pm \sqrt{5^2 - 4(5)0}}{2(5)}$

   C. $x = \frac{-5 \pm \sqrt{15^2 - 4(-5)0}}{2(-5)}$

   D. $x = \frac{-5 \pm \sqrt{5^2 - 4(5)0}}{2(5)}$

3. The learner will calculate the value inside the quadratic formula radical using order of operations.

   The path that a football takes can be described by the equation $h = 15t - 5t^2$ where *h* is the height in meters of the football at time *t*, in seconds. Using the quadratic formula with the coefficients of this situation, what is the value inside the radical?

   A. 245
   B. 225
   C. 205
   D. 15

4. The learner will formulate the two equations from the quadratic formula from the positive and negative algebraic expressions.

   The path that a football takes can be described by the equation $h = 15t - 5t^2$ where *h* is the height in meters of the football at time *t*, in seconds. Use the quadratic formula to solve. What are the two $\pm$ equations for this situation?

   A. $x = \frac{-15+15}{-10}$ and $x = \frac{-15-15}{-10}$

   B. $x = \frac{15+15}{-10}$ and $x = \frac{-15-15}{-10}$

   C. $x = \frac{-15+5}{-10}$ and $x = \frac{-15-5}{-10}$

   D. $x = \frac{-5+15}{-10}$ and $x = \frac{-5-15}{-10}$

5. The learner will simplify a quadratic equation using the quadratic formula and solve for an "x" value.

   The path that a football takes can be described by the equation $h = 15t - 5t^2$ where *h* is the height in meters of the football at time *t*, in seconds. Using the quadratic formula what are the solutions for *x*?

   A. $x = \frac{0}{-10}$ and $x = \frac{-30}{-10}$

   B. $x = \frac{30}{-10}$ and $x = \frac{-30}{-10}$

   C. $x = \frac{-10}{-10}$ and $x = \frac{-20}{-10}$

   D. $x = \frac{10}{-10}$ and $x = \frac{-20}{10}$

6. The learner will express the quadratic solution set, from the equations, using braces { } in the proper order, or indicate "No Solution."

   The path that a football takes can be described by the equation $h = 15t - 5t^2$ where *h* is the height in meters of the football at time *t*, in seconds. Use the quadratic formula to solve for *x.* What is the solution set shown with braces { }? If no solution is found, select "No Solution."

A. {0, 3}
B. {3, 0}
C. {0, 5}
D. No Solution

Comprehensive Goal: Solve a quadratic equation by graphing using a **TI-nspire graphing calculator**.

7. The learner will arrange a quadratic equation in the format $f(x) = ax^2 + bx + c$.

A ball is on the ground and kicked into the air. The upward velocity is 25 meters per second. Assume gravity is measured as 5 meters per second squared. What is the quadratic equation?

A. $f(x) = -5x^2 + 25x$
B. $f(x) = 5x^2 - 25x$
C. $f(x) = -25x^2 + 5x$
D. $f(x) = 25x^2 - 5x$

8. The learner will identify the major calculator buttons and power on the calculator.

To graph a new quadratic equation using a TI-nspire calculator, from the home screen, what is the first menu item you need to select?

A. New Document
B. Current
C. Settings
D. Calculate

9. The learner will create a new document and type the quadratic function in the form "$f1(x) = ax^2 + bx + c$" into the calculator.

To create a new document and graph a new quadratic equation using a TI-nspire calculator, from the home screen, what is the second menu item you need to select?

A. Add Calculator
B. Add Graphs
C. Add Lists
D. Add Data

10. The learner will manipulate the graph of the quadratic function using the “Window / Zoom” function of the calculator.

Enter the quadratic parent function using a TI-nspire calculator and display the graph. Perform one “Zoom In” action with the cursor at the origin. What is the maximum “x” value displayed on the screen?

A. 3
B. 5
C. 7
D. 10

11. The learner will determine the solutions of a quadratic function using the x-intercepts “Zero” function of the calculator.

A ball is on the ground and kicked into the air. The upward velocity is 25 meters per second. Assume gravity is measured as 5 meters per second squared. Enter this function into a TI-nspire calculator. Use the “Zero” function to find the x-intercepts. What are the ordered pair solutions?

A. (0,0); (5,0)
B. (5,0); (0,5)
C. (0,0); (25,0)
D. (5,0); (25,0)

12. The learner will express the quadratic solution set, from x-intercepts, using braces { } in the proper order, or indicate "No Solution."

A ball is on the ground and kicked into the air. The upward velocity is 25 meters per second. Assume gravity is measured as 5 meters per second squared. Enter this function into a TI-nspire calculator. Use the “Zero” function to find the x-intercepts. What is the solution set with braces { }? If no solution is found, select “No Solution.”

A. {0, 5}

B. {5, 0}

C. {0, 25}

D. No Solution
